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**ECON 488/688  Matchings and Markets**

Spring 2021

16:40-19:30 Monday

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COURSE DESCRIPTION

*Matchings and Markets* is about markets where individuals are matched with individuals or firms or items, typically across two sides, as in marriage, employment, university entrance, housing. Such diverse markets are looked at through a common game theoretic model. The aim is to identify outcomes that have nice properties such as stability, optimality or fairness and to design mechanisms to realize such outcomes.

Keywords : competitive cooperative stable outcomes;  existence, optimality, order structures; algorithms, constructive procedures; strategic properties, mechanisms, auctions; institution and market design.

The course is open to all students, in particular from Economics, Computer Science, Industrial Engineering, and Management.

The subject evolved from the original paper by D. Gale and L. Shapley (1962) titled “College Admissions and the Stability of Marriage” (also the paper by M. Beckmann and T. Koopmans (1958) titled “Assignment Problems and the Location of Economic Activities”.)

The Economics Nobel Prize in 2012 was awarded to *Matchings and Market Design* – to two of its original contributors L. Shapley and A. Roth – for remarkable success in the analysis and design of actual matching markets.  Some current ex amples are (1) student placement in schools, (2) labor markets between workers and firms, e.g., doctors and hospitals, (3) organ donation, in which patients are matched to potential donors, (4) rent-controlled housing markets, (5) multi-object auctions. (For an expository newspaper article by Alvin Roth, see [http:/ www.nytimes.com/2014/12/07/nyregion/how-game-theory-helped-improve-new-york-city-high-school-application-process.html?hp&action=click&pgtype=Homepage&module=second-column-region&\_r=0](http://http:/%20%20%20www.nytimes.com/2014/12/07/nyregion/how-game-theory-helped-improve-new-york-city-high-school-application-process.html?hp&action=click&pgtype=Homepage&module=second-column-region&_r=0), and for a video on Kidney Exchange see <http://www.youtube.com/watch?v=dZUfcvBF5K4>.)

The Nobel Prize in Economics in 2020 was awarded jointly to Paul Milgrom and Robert Wilson “for improvements to auction theory and inventions of new auction formats.”

Recently, the subject has also attracted the interest of Computer Science (see the conference series MATCH-UP and *Algorithmic Game Theory*.) Algorithmic complexity and heuristics stands out as a particular theme.

This course is an introduction to the theory of matching and market design. There is no prerequisite. The course does require mathematical thinking and rigor but no specific body of high level mathematical knowledge.

The classical textbook "Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis" by A. E. Roth and Marilda A. O. Sotomayor (1990) and articles to be studied will be uploaded in SuCourse.

COURSE OUTLINE

The first half of the course will be devoted to basic theory :

Matching Theory of one-to-one, one-to-many, and many-to-many markets. Discrete and Continuous Models. Existence, Optimality and Lattice Structure of Stable Allocations; Algorithms and Procedures; Strategyproofness.

(A more detailed outline is in the Appendix below.)

In the second half of the course, various selections from the extended literature will be studied: School Choice, Matching with Contracts, Fair Allocations, Multiobject Auctions, Kidney Exchange, ,…

COURSEWORK AND GRADING

There will be two *exams* one covering the topics in the first half, the other those covered in the second half.

In addition, each student will submit a *term paper* on a topic of her choice approved by the professor. A recommended topic will be integrating “interviewing” into centralized matching systems, for example, in lycee, university or postgraduate level placement in Turkey.

*Attendance* is required.

In determining the letter grade, each exam will have 35% weight, the term paper 20 %, attendance 10%.

APPENDIX

I.  One-To-One Markets

1. (i) The Marriage Model (Gale Shapley 1962) : Existence of stable

matchings; Lattice structure; Optimality. Standard proofs.

                 (ii) The Sellers and Buyers Model with Transferable Utility (Shapley Shubik 1970) and Nontransferable Utility (Alkan Gale 1990) : Existence of stable matchings, equivalence to core allocations and to competitive price equilibria; Lattice structure; Optimality. Proofs elementary and constructive. Multiobject auctions (Demange Gale Sotomayor 1986, Alkan 1992). Link with envyfree allocations of indivisible items (Alkan Demange Gale 1991).

1. Strategyproofness of Optimal Mechanisms (Dubins Freeman 1981, Gale Sotomayor 1985, Leonard 1982)

II. One-To-Many and Many-To-Many Markets

1. To what extent and when the existence-structure-optimality results hold for one-to-many and many-to-many models in Gale Shapley 1962, Kelso Crawford 1982, Roth 1982, 1984, 1985, Gul Stachetti 1999, 2000, Alkan 2002, Fleiner 2003, Alkan Gale 2003, Hatfield Milgrom 2005. A unified look at alternative constructive procedures. Tarski Fixed Point Theorem perspective. The *substitutability* and *size monotonicity* conditions. Comparative statics.

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